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Sonar transducers are usually calibrated prior to installation on a ship. They are not, however, recalibrated after installation even though their receiving (and transmitting) properties can change significantly. In this report we present a procedure for the in situ calibration of sonar transducers at any desired time after installation when the ship is in open water. The procedure provides an up-to-date knowledge of the complex freefield sensitivity (or response) of each of the array transducers. This knowledge can be used to minimize, perhaps even eliminate, the adverse effects on array performance due to changes in the properties of one or more transducers. The in situ calibration procedure is based on a form of three-transducer reciprocity calibration that uses sound propagation factors to account for the influence of the ship's structure. The report describes how to obtain numerical values for the required sound propagation factors. It also describes how to use two-transducer comparison calibration to simplify the in situ calibration process, at least for highly reliable sonars.

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CONTENTS

PROCEDURE FOR THE IN SITU CALIBRATION OF SONAR TRANSDUCERS

I. INTRODUCTION

The receiving (and transmitting) properties of sonar transducers can change with time. This is especially true when the transducers are mounted in a ship's sonar array where they are subject to the harsh ocean environment. A change in the properties of one or more transducers usually translates into a degradation in array performance. The amount of degradation depends on the number of transducers that change and the degree of their change. Fortunately, the degradation in performance can be minimized, perhaps even eliminated, if the changes in the receiving sensitivity (or transmitting response) are compensated for in the array beamformer electronics. However, this requires an up-to-date knowledge of the complex sensitivity (or response); i.e., both the amplitude and the phase angle, for each of the array transducers.

Although transducers are often calibrated prior to installation on the ship, at present they are not recalibrated after installation. In this report we present a procedure for the in situ calibration of sonar transducers at any desired time after installation when the ship is in open water. It can be used to obtain the complex freefield receiving voltage sensitivities (and, if desired, the transmitting current responses) of all of the transducers in the sonar array. The heart of the procedure is in situ reciprocity calibration. This is a form of three-transducer reciprocity calibration that uses sound propagation factors to account for the influence of the ship's structure. We develop the theory for in situ reciprocity calibration in Section II of this report. In Section III, we describe how to obtain empirical values for the unknown sound propagation factors. Section IV considers two-transducer comparison calibration and explains how to use it to simplify the in situ

calibration process in exchange for a possible small loss in accuracy. We conclude the report in Section V with a step-by-step description of our recommended procedure for in situ calibration.

II. IN SITU RECIPROCITY CALIBRATION

In situ reciprocity calibration is a generalization of three-transducer freefield reciprocity calibration to include non-freefield conditions. A comprehensive description of freefield reciprocity calibration (without phase) is given by Bobber [1] while the extension to include phase is described in Ref. 2. In the present analysis, we consider any three transducers, T_1, T_2, and T_3, in the sonar array. We use the term "transducer" here to mean either a single element or a cluster (e.g., stave) of elements that are joined electrically and treated as a single element for beamforming purposes. We assume that each transducer is reciprocal and can either be driven as a projector or receive sound as a hydrophone. This assumption requires that any preamplifiers, automatic gain control circuits, or other nonreciprocal electrical components that are connected to the transducers be disconnected during the reciprocity calibration measurements. The electronic calibration of nonreciprocal components can be performed, when required, by using conventional electrical measurements.

A series of three projector-hydrophone measurements, as shown in Table 1, is required to determine the freefield voltage sensitivity (and, from reciprocity, the transmitting current response) of the three transducers. Typically, one first drives T_1 and receives with T_2, then drives T_1 and receives with T_3, and lastly drives T_3 and receives with T_2.

Table 1 - Measurements for three-transducer reciprocity calibration.

MEASUREMENT SETUP NO.	INPUT CURRENT	PROJECTOR	HYDROPHONE	OUTPUT VOLTAGE
1	i_{12}	T_1	T_2	e_{12}
2	i_{13}	T_1	T_3	e_{13}
3	i_{32}	T_3	T_2	e_{32}

Let us consider the first of these measurements. The current used to drive T_1 is i_{12}. The acoustic pressure p_{12} produced by T_1 in the vicinity of T_2 is given by $S_1 i_{12} \beta_{12}$, where S_1 is the transmitting current response of T_1 and the sound propagation factor β_{12} is a dimensionless, frequency dependent, complex quantity that is a function of the sonar and ship construction and the locations of both T_1 and T_2. If T_2 was in the farfield of T_1 and the measurement involved only the two transducers in open water; i.e., without the rest of the sonar and ship present, then β_{12} would be given by

$$\beta_{12} = \left[\frac{d_o}{d_{12}}\right] \exp[jk(d_o - d_{12})] \quad . \tag{1}$$

Here the distance d_0 is usually equal to 1 m as specified in the definition of S_1, and the distance d_{12} is measured between specified reference points on T_1 and T_2. The wavenumber k is given by ω/c, where ω is the angular frequency in radians per second and c is the sound speed in the surrounding medium (usually seawater) in meters per second. The assumed time dependence $\exp(j\omega t)$ has been suppressed for convenience. For the in situ case, however, the situation is much more complicated and a corresponding closed form for β_{12} is unlikely to be available. For the present, we assume that we have numerical values for both the amplitude and phase angle of β_{12}. We note that the sound propagation factor is usually reciprocal; i.e., β_{12} is equal to β_{21} or, in general, β_{ij} is equal to β_{ji}. Thus the propagation factor is unchanged when the roles of the transducers are interchanged so that transducer T_j becomes the projector and T_i becomes the receiver. The reciprocity of β follows from the assumed reciprocity of the appropriate Green's function. If the Green's function is not reciprocal; e.g., if the effects of flow due to motion of the sonar platform through the water require a nonreciprocal Green's function, then β is not reciprocal and β_{ij} and β_{ji} must be treated as independent quantities. To avoid this possibility, we recommend that the in situ calibration be performed at platform speeds less than a few knots until test measurements are made that demonstrate the reciprocity of β for higher speeds. [See the comments regarding such a test following Eq. (16)].

The open-circuit voltage produced by T_2 is given by

$$e_{12} = M_2 p_{12} = M_2 S_1 i_{12} \beta_{12} \quad , \tag{2}$$

where M_2 is the receiving voltage sensitivity of T_2.

Similarly, we obtain for measurement setup 2

$$e_{13} = M_3 p_{13} = M_3 S_1 i_{13} \beta_{13} \quad , \tag{3}$$

where M_3 is the receiving voltage sensitivity of T_3. In freefield reciprocity calibration, one often chooses the drive current i_{13} for T_1 in this measurement to be equal to that used in the first measurement. However, this choice might not be desirable if one of the hydrophones T_2 or T_3 is much less sensitive than the other.

From setup 3 we obtain

$$e_{32} = M_2 p_{32} = M_2 S_3 i_{32} \beta_{32} \quad . \tag{4}$$

Equation (4) can be rewritten in terms of M_3 instead of S_3 by using $M_3 = JS_3$, the relationship that is satisfied by reciprocal transducers. Here J is the complex spherical wave reciprocity parameter given by Beranek [3]:

$$J = \left[\frac{4\pi d_0}{j\omega\rho}\right] \exp(jkd_0) \quad , \tag{5}$$

where ρ is the density of the surrounding medium. Equation (4) now becomes

$$e_{32} = \frac{M_2 M_3 i_{32} \beta_{32}}{J} \quad . \tag{6}$$

Combining Eqs. (2), (3), and (6), we obtain the following expression for the receiving voltage sensitivity of transducer T_2:

$$M_2 = \left[\frac{e_{12} e_{32} i_{13} \beta_{13} J}{e_{13} i_{12} i_{32} \beta_{32} \beta_{12}}\right]^{1/2} \quad . \tag{7}$$

Substitution of M_2 into Eq. (6) produces

$$M_3 = \left[\frac{e_{13} e_{32} i_{12} \beta_{12} J}{e_{12} i_{13} i_{32} \beta_{32} \beta_{13}}\right]^{1/2} \quad . \tag{8}$$

Substitution of M_2 into Eq. (2) with the use of the reciprocity relationship $M_1 = JS_1$ results in

$$M_1 = \left[\frac{e_{12}e_{13}i_{32}\beta_{32}J}{e_{32}i_{12}i_{13}\beta_{12}\beta_{13}}\right]^{1/2} . \qquad (9)$$

The sensitivity values obtained from Eqs. (7) through (9) are complex quantities; they possess both an amplitude and a phase angle. The amplitude $|M|$ is found from the amplitudes of the measured voltages and currents; i.e.,

$$|M_1| = \left[\frac{|e_{12}||e_{13}||i_{32}||\beta_{32}||J|}{|e_{32}||i_{12}||i_{13}||\beta_{12}||\beta_{13}|}\right]^{1/2} , \qquad (10)$$

while the phase angle $\Phi(M)$ is obtained from the corresponding phase angles Φ; e.g.,

$$\Phi(M_1) = \frac{1}{2}\,[\Phi(e_{12}) + \Phi(e_{13}) + \Phi(i_{32}) + \Phi(\beta_{32}) + \Phi(J) - \Phi(e_{32}) - \Phi(i_{12}) - \Phi(i_{13}) - \Phi(\beta_{12}) - \Phi(\beta_{13})] . \qquad (11)$$

Here $\Phi(z)$ denotes the phase angle $\tan^{-1}(y/x)$ of the complex quantity $z = x + iy$. We note that the transducer transmitting current responses S_j can readily be obtained, if desired, from the sensitivities M_j by use of the reciprocity relationship $S = M/J$.

The sensitivities of all of the transducers in a sonar array can be obtained, three transducers at a time, by use of Eqs. (7) through (11). However, evaluation of these equations requires numerical values for the propagation factors β_{12}, β_{13}, and β_{32}. In the next section, we describe how to obtain these values empirically.

III. EVALUATION OF SOUND PROPAGATION FACTORS

Calculation from first principles of the sound propagation factors β_{ij} for ship-mounted sonar systems would be extremely difficult, if not

impossible. Therefore, we propose an empirical solution to the problem based on measurements. Equations (2) through (6) can be solved for β_{12}, β_{13}, and β_{32}. We obtain

$$\beta_{12} = \frac{e_{12}J}{M_1 M_2 i_{12}} , \tag{12}$$

$$\beta_{13} = \frac{e_{13}J}{M_1 M_3 i_{13}} , \tag{13}$$

$$\beta_{32} = \frac{e_{32}J}{M_2 M_3 i_{32}} , \tag{14}$$

or, in general,

$$\beta_{ij} = \frac{e_{ij}J}{M_i M_j i_{ij}} . \tag{15}$$

From the reciprocity of β_{ij}, we also have

$$\beta_{ij} = \beta_{ji} = \frac{e_{ji}J}{M_i M_j i_{ji}} . \tag{16}$$

Consequently, we can determine the factors β_{ij} from a projector-hydrophone measurement either with T_i as a projector and T_j as a hydrophone or with T_j as a projector and T_i as a receiver. We note that the reciprocity of β can be tested by comparing the values obtained for β_{ij} from the two different measurements.

The sensitivities M_i and M_j required in Eq. (16) can be obtained by freefield measurements made in a calibration facility immediately before installation of the transducers on the ships. The measurements to determine β_{ij} are then made as soon as possible after ship installation in order to minimize the possibility of changes in the sensitivities from the pre-installation values. For an array with N transducers, there are N(N-1)/2 independent factors β_{ij}, $i=1,2...N-1; j=1,2,...N; j>i$. As we will see later in Section V, we do not need values for all of these factors but only for a relatively small number of them. Once the required factors β_{ij} are known, the transducer sensitivities can be determined at any later time by the use of

in situ calibration. This assumes, of course, that the factors β_{ij} remain unchanged; i.e., that there are not significant changes in the acoustic boundary conditions associated with sound propagation between each of the relevant pairs of transducers in the sonar array. The acoustic boundary conditions depend on both the geometry of the sonar array and neighboring structures and the input mechanical impedance of the array transducers. We assume that there are no significant changes in the geometry. Changes in input mechanical impedance can accompany changes in transducer sensitivity (or transmitting response). However, we do not expect these changes to significantly affect the propagation factors β_{ij}, except possibly at frequencies near transducer mechanical resonance and then only for a substantial loss of sensitivity (or transmitting response) of transducers located directly between the i^{th} and j^{th} transducer. If in situ calibration is desired for frequencies near mechanical resonance, measurements can be made for the given sonar to determine whether the factors β_{ij} change significantly when neighboring transducers severely degrade.

IV. IN SITU COMPARISON CALIBRATION

The three-transducer reciprocity procedure described in Section II can be used to calibrate all of the transducers in the array, three transducers at a time. However, it may be simpler (at least for highly reliable sonars) to calibrate only a few selected master transducers throughout the array using the reciprocity procedure and then to use these transducers to calibrate the remaining transducers by two-transducer comparison calibration. The price for using comparison calibration is a possible reduction in the accuracy of the sensitivity values. The accuracy may still be acceptable for operational purposes. Future studies will be required to make this determination.

In situ comparison calibration involves measurements similar to those of setups 1 and 2 in Table 1. First we drive one of the master transducers as the projector T_1 and receive with another master transducer as the hydrophone T_2 shown in setup 1. (This measurement may already have been made during the three-transducer reciprocity calibration of T_1 and T_2.) We then make a measurement using setup 2 with one of the uncalibrated transducers as hydrophone T_3. Combining Eqs. (2) and (3) results in the following expression that can be used to compute the unknown sensitivity M_3 from the measured voltages e_{12} and e_{13} and currents i_{12} and i_{13}:

$$M_3 = M_2 \frac{\beta_{12} e_{12} i_{13}}{\beta_{13} e_{13} i_{12}} \quad . \tag{17}$$

The sensitivity of each of the other uncalibrated transducers can then be obtained by repeating setup 2 measurements with different choices for T_3. When a different master transducer is used as the projector T_1 (and probably a different master transducer used for the hydrophone T_2), we must make a setup 1 measurement for these transducers, assuming the measurement has not already been made.

V. RECOMMENDED PROCEDURE FOR IN SITU CALIBRATION

The following is our recommended procedure for the in situ calibration of transducers in a sonar array.

1. *All of the transducers are calibrated prior to ship installation by freefield measurements made in a calibration facility such as NRL-USRD.* We recommend that three-transducer reciprocity calibration be used for this purpose. Reference 2 describes reciprocity calibration measurements with phase included. If reciprocity calibration is not feasible, for reasons of time or cost, two-transducer comparison calibration using a single standard hydrophone as a reference is acceptable. The extra uncertainty introduced into the sensitivity values by comparison calibration is essentially the uncertainty in the sensitivity of the reference hydrophone. Since this extra uncertainty is common to all of the comparison calibration measurements, it cancels when we form ratios of the resulting transducer sensitivities. Hence, relative sensitivities obtained using comparison calibration are as accurate as those obtained using reciprocity calibration. Beamforming accuracy, which depends on relative rather than absolute transducer sensitivities, will not be compromised by the use of comparison calibration.

   If each of the transducers is actually a cluster of hydrophones that is joined electrically and treated as a single element for beamforming purposes, then the pre-installation calibration of each

cluster should be performed with the cluster hydrophones connected electrically and arranged geometrically the same as they are in the sonar array. The resulting freefield sensitivities will represent clusters rather than individual hydrophones. Often, however, the electrical leads of the hydrophones are individually accessible on board the ship; e.g., at the input to a junction box where they are connected electrically to form a hydrophone cluster. In this case we can choose to calibrate on an individual hydrophone rather than a cluster basis. Although doing so significantly increases the number of sensitivity values and propagation factors that must be determined and retained, the additional information can be very valuable for diagnostic and beamforming purposes.

2. *The choice is made between case (a) using in situ reciprocity calibration for all of the transducers in the sonar array and case (b) using in situ reciprocity calibration for selected master transducers and using these, via in situ comparison calibration, to determine the sensitivities of the remaining transducers in the array.*

3. *If the choice in step 2 above is case (a), then the three-transducer combinations; i.e., triads, involved in the reciprocity calibrations are selected.* The selection process involves trial and error; it needs only be done once for each type of sonar array. Each triad should be chosen so that its three transducers are as close as possible in order to maximize the magnitudes of the received signals acquired during the calibration process relative to the acoustic signals simultaneously being radiated to the ship's farfield. This requirement assumes that maintaining ship covertness during the calibration measurements is important. Covertness should be reasonably easy to maintain since acoustic signals just sufficient for in situ calibration measurements (made very close to the projector) will be weak and difficult to detect in the farfield of the ship. If the hydrophone and/or cable shielding does not provide sufficient electromagnetic suppression, thetransducers in each triad must also be located far enough apart to allow suitable signal gating to avoid electromagnetic interference. (We assume that the calibration measurements are made using pulsed sinusoids

with pulse lengths sufficient to ensure steady-state results.) When electromagnetic interference is negligible, the triads can be near-neighbor combinations. For example, if the transducers are numbered in order of their location from one side of the array to the other, then triad 1 would include transducers 1, 2, and 3; and, in general, triad n would include transducers 3(n−1) + 1, 3(n−1) + 2, and 3(n−1) + 3.

4. *If the choice in step 2 above is case (b), then the master transducers are selected.* These transducers should be scattered throughout the sonar array. There should be enough of them so that every sonar transducer is reasonably close to at least one of the master transducers, and so that they can be divided for reciprocity calibration into triads of reasonably close transducers. For example, in the array mentioned above in step 3 one might select transducers 3, 8, 13, 18, etc. as the master transducers with 3, 8, and 13 forming the first master triad, etc.

5. *The sound propagation factors are obtained as soon as possible after installation of the transducers in the sonar array.* The required factors β_{ij} are determined by projector-hydrophone measurements either with transducer T_i as projector and T_j as hydrophone [Eq. (15)] or vice versa [Eq. (16)]. However, before we make measurements, we must first determine which of the N(N−1)/2 independent sound propagation factors (for an array with N transducers) are required.

For case 2(a), we need values only for the factors linking the transducers in each triad. In the example given above in step 3, this includes β_{12}, β_{13}, β_{23}; β_{45}, β_{46}, β_{56} etc., for a total of N propagation factors. (Actually, if N is not a multiple of three, the number of factors that are required will be one or two greater than N.) The need for only N propagation factors rests on the assumption that all three transducers in each triad retain sufficient sensitivity (and response) such that there is no significant reduction in signal-to-noise for any of the three in

situ calibration measurements. This assumption is unlikely to be met in practice. When the sensitivity (and response) of one of the transducers decreases to a point that the accuracy of one (or two) of the received voltages e_{ij}, e_{ik}, or e_{jk} is reduced, then the accuracy of <u>all three</u> resulting sensitivity values M_i, M_j, and M_k as obtained using Eqs. (7) through (9) will also be reduced. To protect against this situation, we build redundancy into the calibration process. In particular, we divide the array into groupings of more than three transducers. As long as at least three transducers in each grouping retain adequate sensitivity (and response), these "good" transducers can be calibrated by reciprocity measurements involving only "good" transducers. The sensitivity of each of the other; i.e., degraded, transducers can then be obtained by reciprocity measurements involving two of the "good" transducers and the degraded transducer. Although the accuracy of the resulting sensitivity values for the degraded transducers will probably be reduced, accuracy for the "good" transducers will remain undiminished. We note that the term "degraded" as used here does not mean that the transducer is nonfunctioning and should be disconnected. It merely means that the sensitivity (or response) has dropped to the point that there is a significant reduction in the signal-to-noise ratio for projector-hydrophone calibration measurements involving the degraded transducer. Of course, the transducer could be so severely degraded that use of the transducer as a receiver, together with amplification of its output to account for reduced sensitivity, results in diminished sonar performance. Similarly, its use as a projector might be impossible because of an inability of the sonar drive electronics to provide the required amplification. In this case, the sonar operator should disconnect the transducer (or adjust its shading to zero) and reshade the remainder of the sonar transducers to provide optimum array performance.

Selection of the number of transducers N_g to include in each grouping depends on the percentage of transducers in the array that are expected to degrade before the array is retrofitted. If the percentage is moderately low; e.g., below 15 percent, then a very

conservative choice for N_g would be six. The likelihood that at least three of the transducers would remain "good" would be nearly unity [4]. With N_g equal to six, however, we must determine and retain two and one-half times as many sound propagation factors as when N_g is equal to three. For the example given above in step 3, we require values for β_{12}, β_{13}, β_{14}, β_{15}, β_{16}, β_{23}, β_{24}, β_{25}, β_{26}, β_{34}, β_{35}, β_{36}, β_{45}, β_{56}, etc. Thus we now need 15 factors for the first 6 transducers as opposed to 6 factors before. If we increase the redundancy further by making N_g equal to nine, we require four times as many factors as when N_g is equal to three. An approximation for the number of factors that are required is given by $N(N_g-1)/2$. This expression is exact of N and N_g are multiples of three.

For case 2(b), we need values for both the factors linking the transducers in each master triad and the factors linking each non-master transducer to one of the master transducers. For the example given above in step 4, this includes β_{38}, $\beta_{3.13}$, $\beta_{8,13}$, β_{13}, β_{23}, β_{34}, β_{35}, β_{68}, β_{78}, etc., for a total of exactly N propagation factors, essentially the same as for case 2(a). When we add redundancy for case 2(b), however, the situation becomes more complicated. Consider an increase in the number of potential master transducers for each non-master transducer in order to protect against reduced accuracy in both the sensitivity of the master transducers obtained by reciprocity calibration and the sensitivity of the remaining transducers obtained by comparison calibration. If, for example, we consider the example given above in step 4 and increase the redundancy by a factor of two by providing two potential master transducers for each non-master transducer, we then require values for the factors β_{38}, $\beta_{3,13}$, $\beta_{3,18}$, $\beta_{3,23}$, $\beta_{3,28}$, $\beta_{8,13}$, $\beta_{8,8}$, ..., $\beta_{23,28}$, β_{13}, β_{23}, β_{34}, β_{35}, β_{18}, β_{28}, β_{38}, β_{48}, β_{58}, etc., for a total of approximately 2N factors. Although this is comparable to the number of factors required for case 2(a) when we double the grouping size, it does not provide near the same protection. In case 2(a) it is only necessary for three out of six transducers to remain "good." In case 2(b) it is necessary that one out of two master transducers remain "good," a much more stringent

requirement. We have to increase the redundancy of the master transducers in case 2(b) by a factor of 20 in order to equal the protection obtained for case 2(a) by increasing the transducer groupings to six. This would increase the number of required propagation factors for case 2(b) significantly beyond that required for case 2(a). Thus, when significant redundancy is required, we do not recommend the use of in situ comparison calibration because of the large number of propagation factors that must be determined and retained. In situ comparison calibration appears to be suitable primarily for sonar arrays that are very reliable, i.e., for which relatively few transducers degrade before the sonar is retrofitted.

6. *The in situ calibration is performed using the procedure for either case 2(a) or 2(b).* The operator and/or controlling computer software should examine the sensitivity values as they are obtained in order to determine when one of the transducers has degraded. Assuming that he has values for the additional propagation factors that are required, the operator can then substitute a "good" transducer for the degraded transducer during subsequent calibration measurements to determine the sensitivities of the other transducers. This substitution will eliminate the loss in accuracy of the sensitivity obtained for "good" transducers due to the use of degraded transducers in their calibration.

VI. CONCLUSION

The procedure described in this report can be used to determine the up-to-date sensitivity (and response) of all of the transducers in a sonar array. The resulting values are freefield values; otherwise, they could not be compared to pre-installation values obtained from measurements made in a calibration facility such as NRL-USRD. Beamforming based on pre-installation sensitivities can be modified during a sea exercise to reflect changes in both the amplitude and phase of the transducer sensitivies (and responses). This modification can minimize, perhaps even eliminate, the otherwise deleterious effects due to changes in transducer sensitivities (and responses).

Implementation will probably require a significant increase in the sophistication of the sonar system, but the operational advantages are obvious.

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3. Beranek, L.L., *Acoustic Measurements*, John Wiley and Sons, Inc., New York, N.Y., 1949, p 120.

4. The probability that at least three out of N_g transducers will remain "good" when the probability that a given transducer will remain "good" is equal to μ is given by

$$\sum_{n=0}^{3} N_g!(1-\mu)^n(\mu)^{N_g-n}/[n!(N_g-n)!] \quad .$$

For $\epsilon = 0.15$ and $N_g = 6$, we obtain a probability of 99.4%.